Sinusoidal Regression (3.7.A.4) Homework

1. On the morning of November 7, 1940, a strong wind hit the Tacoma Narrows Bridge, and the bridge began swinging up and down. Eventually, the bridge collapsed. Incredibly the owners of a local camera shop happened to have filmed the bridge. Engineers study the footage to learn from it and prevent future bridge collapses. Below is data on the height of a point on the bridge (measured in feet above the Puget Sound) t seconds after filming began.

t	0	3	6	9	12	15	18
$h(t)$	173.9	215.1	201.5	182.3	227.1	173.9	215.1

   a. The height of that point on the bridge can be modelled by $f(t) = a\sin(bx + c) + d$. Use the regression capabilities of Desmos to find a, b, c, and d.

   b. According to the model, what is the maximum height that this point on the bridge reached.

   c. According to the model, how much time passed between one moment the point was at its maximum height to the next moment the bridge was at its maximum height.

2. From 2004 to 2009, the Taipei 101 skyscraper in Taiwan was the tallest building on earth. Being so tall, it faces strong forces from the wind. Further, the building is located near a major fault line, so earthquakes and tsunamis are a concern. To counteract the swaying of the building, a 660 metric ton tuned mass damper is situated near the top of the building. As the building sways, the damper counteracts that motion, protecting the structure from stress. During the 2008 Sichuan earthquake, the damper was recorded swaying. Below is data on the speed of the damper, measured in meters per second, t seconds after recording began during the earthquake.

t	0	1	2	3	4	5	6	7
$s(t)$	0.577	1.863	3.063	0.008	2.235	1.731	0.551	3.467

   a. The speed of the damper can be modelled by a sinusoidal function $g(t) = a\sin(bx + c) + d$. Use the regression capabilities of Desmos to find a, b, c, and d.

   b. According to the model, what is the period of the damper's oscillations in speed? What is the frequency?

   c. According to the model, what is the speed of the damper at time $t = 4$? How far off is the model from the actual speed at time 4 given in the table?

   d. Based on the model, how many times during the first ten seconds after the earthquake was the speed of the damper 2 meters per second?

3. Sunspots are temporary phenomena on the Sun's photosphere that appear as spots darker than the surrounding areas. Sunspots are related to solar weather and are an important topic of study for both scientists interested in space weather and scientists interested in protecting earth's electrical infrastructure from strong solar flares. Below is data on the average daily number of sunspots observed over a fifteen year period. (Source: WDC-SILSO, Royal Observatory of Belgium, Brussels). The year is given in "years since 2000". There were too many data points to fit in one row, so it looks like two tables. It is all one table, just continued on a new line.

Year	8	9	10	11	12	13	14	15
Avg Sun Spots	4.2	4.8	24.9	80.8	84.5	94	113.3	69.8

Year	16	17	18	19	20	21	22
Avg Sun Spots	39.8	21.7	7	3.6	8.8	29.6	83.2

a. Use the regression capabilities of Desmos to construct a sinusoidal model of the average daily number of sunspots observes as a function of the year.

b. As of the writing of this question, Wikipedia says sunspots follow a "approximately 11-year solar cycle." Do the data support that? Based on the data from these fifteen years, what is the period of the sunspot cycle? Round to one decimal place.

c. Use the model to predict the average daily number of sunspots in 2023.

d. The actual daily average number of sunspots in 2023 was 125.5. By how much was the model off?

4. On a given night, not all of the moon is visible. Below is data reflecting what percent of the moon was visible at midnight from Weston, FL every third night in the month of November, 2023. All data is given in terms of midnight standard time, not daylight savings time. (source: US Navy Astronomical Applications Department Website aa.usno.navy.mil/data/MoonFraction)

Day of Month	1	4	7	10	13	16	19	22	25	28
Fraction Illuminated	87%	61%	33%	10%	0%	9%	36%	70%	94%	99%

a. The fraction of the moon illuminated at midnight in Weston as a function of the day of the month in November can be modelled by a sinusoidal function $f(t) = a\sin(bx + c) + d$. Use the regression capabilities of Desmos to find a, b, c, and d.

b. Based on the context of this question, what values of a and d should one reasonably expect?

c. Based on the model, what is the period of the moon's visibility cycle?

d. If this same question were completed with data from August 1992, which parameter in the function would most likely be different: a, b, c, or d?

e. There are thirty days in November. How can the value of $f(31)$ be interpreted?

While you have Desmos open, let's do a little more practice with Desmos:

5. Let $f(x) = \frac{1}{3}x^3 - x^2 + 1.24x - 4$
   a. Find $f(3.1)$
   b. Find $f^{-1}(2.3)$
   c. Find $(f \circ f)(0.8)$
   d. Find all input values in the domain of f that yield an output value of 5

6. Let $h(x) = 5\sin\left(\frac{\pi}{4}x\right) - 3\cos\left(\frac{\pi}{2}x\right)$.
   a. Use the location of relative maxima on the graph to determine the period of h.
   b. Use the values of the maximum and minimum of the function to determine the amplitude of h.

7. Let $k(x) = x^4 - 4x^2 + 3x + 1$.
   a. Find the absolute minimum value of $k(x)$.
   b. At what x-values does $k(x)$ have relative minima?
   c. Over what interval or intervals of input values is $k(x) > 4$?

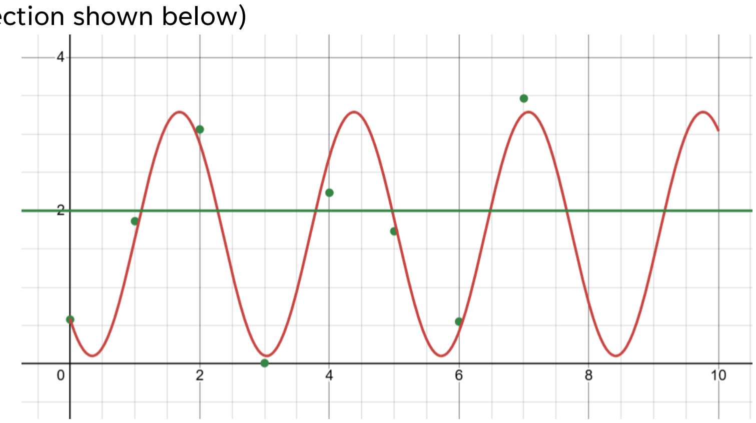
Sinusoidal Regression HW Answers

(note: your answers should match the answer key to at least 3 decimal places. If we disagree in a less significant decimal place, do not worry)

1.
   a. $a = 27.985144$
      $b = 0.837668$
      $c = -1.940481$
      $d = 199.97895$
   b. 227.964 feet
   c. This is a question about the period of the model.
      Period $= \frac{2\pi}{b} = 7.501$ seconds (any answer between 7.497 and 7.506 is close enough)

      To the left is a clickable/scannable link to a video showing the collapse of this bridge. We should take interest in events like this not because we delight at destructive moments in which people get injured but rather so that we can understand the math to become engineers who keep people safe in the future.

2.
   a. $a = 1.59397$
      $b = 2.33375$
      $c = -2.36376$
      $d = 1.69582$
   b. Period $= \frac{2\pi}{b} = 2.692 \frac{\text{second}}{\text{cycle}}$.
      Frequency $= \frac{1}{\text{Period}} = \frac{b}{2\pi} = 0.371 \frac{\text{cycles}}{\text{second}}$.
   c. $g(4) = 2.708 \frac{\text{m}}{\text{s}}$. The model's prediction is $0.473 \frac{\text{m}}{\text{s}}$ more than the actual value of the speed of the pendulum at time $t = 4$.
   d. 7 times (each intersection shown below)



To the left is a clickable/scannable link to a video showing Taipei 101's tuned mass damper swaying during an earthquake, though I do not believe it is the earthquake that this question relates to.

3.

a. $f(t) = a\sin(bx + c) + d$
   $a = 52.86882$
   $b = 0.584269$
   $c = 0.123784$
   $d = 50.58834$
b. Period $= \frac{2\pi}{b} = 10.754$. It appears the model is relatively consistent with Wikipedia's statement.
c. $f(23) = 94.950$. The model predicts an average daily number of sunspots of 94.950 sunspots in the year 2023.
d. The model underestimated by 30.550

4.

a. $a = 0.498546$
   $b = 0.210303$
   $c = 2.03835$
   $d = 0.4970644$
b. The fraction of the moon illuminated should be at most $100\% = 1$ and should have a minimum value of $0\% = 0$. So, a good model should have a midline of $50\% = 0.5$ and an amplitude of $50\% = 0.5$. $d = 0.5$ (because midline) and $a = 0.5$ (because amplitude). Wow, those are pretty much the values Desmos gave! Math works! I thought it was all just made up like unicorns and platypuses, but now I see math is real!
c. Period $= \frac{2\pi}{b} = 29.877$ days (anything between 29.874 and 29.920 is close enough). That's close to the value that I think is the actual value of the period of the moon cycle!
d. c would likely change the most dramatically. The amplitude and the midline of the moon cycle were the same in 1992 as they are today, so a and d should not change. The period of the moon was also the same in 1992 as it is today, so b should not change. But it's likely that on August 1st 1992, the moon was at a different phase in its cycle.
e. $f(31)$ is the predicted visibility of the moon on December 1st 2023.

5.

a. 0.164
b. 3.487
c. -34.420
d. $x = 3.853$

6.

a. 8
b. 6.021

7.

a. -7.494
b. $x = -1.574$ and $x = 1.164$
c. $(-\infty, -2.402) \cup (1.805, \infty)$